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Reg. No. : .....

Name : .....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1131.1 : MATHEMATICS – I – CALCULUS WITH APPLICATIONS IN PHYSICS – I

(2018-2020 Admission)

Time : 3 Hours

Max. Marks : 80

PART – I

Answer **all** questions. Each question carries **1** mark.

1. Find the derivative of $f(x)=\frac{\sin x}{x}$ with respect to x.

2. Stationary point of inflection a function $f(x)$ is a point at which ————————

3. State Rolle's theorem.

4. The area of a circle with radius a is ————————

5. Define the mean value of a function.

6. Define absolute convergence.

7. State D'Alembert's ratio test.

P.T.O.

8. If $v = -2i + k$, $w = 3i + 5j - 4k$ then find $v + w$.

9. Find the magnitude of the vector $2i + 3j + 6k$.

10. If the vectors a, b and c are coplanar, then $a \cdot (b \times c) = ...$

(10 × 1 = 10 Marks)

PART – II

Answer any **eight** questions. Each question carries **2** marks.

11. Find the derivative with respect to x of $f(t) = 2at$, where $x = at^2$.

12. Find the derivative with respect to x of $f(x) = x^3 \sin x$.

13. Evaluate $\int x \sin x \, dx$.

14. Evaluate the integral $I = \int_0^\infty \frac{x}{(x^2 + a^2)^2} \, dx$.

15. Find the volume of a cone enclosed by the surface formed by rotating about the x-axis the line $y = 2x$ between $x = 0$ and $x = h$.

16. Use Leibnitz' theorem to find the third derivative of the function $x^3 \sin x$.

17. Find Sum the integers between 1 and 200 inclusive.

18. Define interval of convergence of a power series and find the interval of convergence of the power Series $\sum_{k=0}^{\infty} \frac{x^k}{k!}$.

19. Find the Maclaurin series for $f(x) = \cos x$.

20. Determine whether the following series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$ converges.

21. Two particles have velocities $v_1 = i + 3j + 6k$ and $v_2 = i + 3j - 2k$, respectively. Find the velocity of the second particle relative to the first.

22. Find the area of the parallelogram with sides $a = i + 2j + 3k$ and $b = 4i + 5j + 6k$.

(8 × 2 = 16 Marks)

PART – III

Answer any **six** questions. Each question carries **4** marks.

23. Using logarithmic differentiation, find the derivative of $y = x^x$.

24. Find the positions and natures of the stationary points of the function $f(x) = 2x^3 - 3x^2 - 36x + 2$.

25. Find the radius of curvature of $x^2 + y^2 = 1$.

26. Evaluate the integral $\int \frac{2}{1+3\cos x} dx$.

27. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 2$.

28. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)}$.

29. Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converges.

30. Find the angle between the vectors $a = i + 2j + 3k$ and $b = 2i + 3j + 4k$.

31. Find the minimum distance from the point P with coordinates (1, 2, 1) to the line $r = a + \lambda b$, where $a = i + j + k$ and $b = 2i - j + 3k$.

(6 × 4 = 24 Marks)

PART – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) State Mean Value Theorem. **9**

    (b) Determine inequalities satisfied by

        (i) $\ln x$ and

        (ii) $\sin x$ for suitable ranges of the real variable x. **6**

33. (a) Evaluate the integral $I = \int \frac{1}{\sqrt{1-x^2}} dx$. **5**

(b) Show that the value of the integral $I = \int_0^1 \frac{1}{\left[1+x^2+x^3\right]^{3/2}} dx$ lies between 0.810 and 0.882. **10**

34. (a) Find the sum the series $S = 1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots$ **5**

(b) Find the power series expansion of $e^{\cos x}$. **10**

35. (a) Find the direction of the line of intersection of the two planes $x + 3y - z = 5$ and $2x - 2y + 4z = 3$. **6**

(b) Find the distance from the point P with coordinates (1, 2, 3) to the plane that contains the points A, B and C having coordinates (0, 1, 0), (2, 3, 1) and (5, 7, 2). **9**

(2 × 15 = 30 Marks)
